Forms of Reasoning

Based on discussions this week, the following types of justifications seem to be important:

- **Model-Based Justification**: The student uses a (presumably) agreed-upon model to justify their result. Examples of such models could include different models for operations like combining (addition), take-away (subtraction), or the area model (multiplication). It could also be a model for a kind of quantity (like a group of circles to represent a whole number). It could also be more complex (like the way we use tree diagrams to factor or to do counting problems).

  This kind of reasoning can be used in both valid and invalid justifications. Issues include:

  - Does the model really capture the general case (if the statement to prove is general)?
  - Is the model used correctly?

  As we have seen, it is also very important that the audience understands what model is being used so they can follow the justification!

  Example of this form of reasoning: Heather (the Green sheet) used this approach in showing a comparison model (using squares instead of people, and consistently using a comparison model) to justify the claim that 4 ½ - 1 ½ = 3.

- **Proving an Equivalent Statement**: The student proves the original claim by instead showing that another equivalent statement is true.

  This kind of reasoning can be used in both valid and invalid justifications.. Issues include:

  - Is the alternative statement actually equivalent?
  - Is the justification of the alternative statement valid?

  Another big issue with this form of reasoning is that the audience needs to understand that the student is trying to prove an equivalent statement AND understand that it is equivalent.

  Example of this form of reasoning: Isaac (the Yellow sheet) used this approach (using a combining model for addition) to show that 4 ½ - 1 ½ = 3 by instead justifying the claim that 3 + 1 ½ = 4 ½. [This also illustrates that different forms of reasoning can be used together.]

- **Example Based Reasoning**: There are actually two very different forms of reasoning that fit in this category…

  3 (a) **Empirical Reasoning**: A student concludes from an example or a collection of examples that the statement must always be true. This is only valid if all possible examples are checked!

  3(b) **Generic Example:** A student uses a specific example BUT reasons about it in a general way. For example they may focus on the structural aspects of a specific figure and argue about WHY it works in a way that makes it clear that it will always work.

  Note that a high quality argument using a generic example can often be used to construct a general proof by replacing the specific example with a representation of the general case.

- **Pattern Based Reasoning**: There are three different forms that fit in this category.

  4(a) **Pattern *Result* Reasoning**: A student notices a pattern in the results and assumes it must be the correct pattern and uses this assumption to justify the general case (perhaps using symbols). This is a variant on Empirical Reasoning and is not valid by itself (it needs to evolve into one of the next two sub-types).

  4(b) **Pattern *Process* Reasoning**: A student notices a pattern and then figures out why the pattern must hold. OR a student analyzes a process and figures out that a given pattern must be produced. Then based on this, the student justifies the claim about the general case (perhaps using symbols).

  4(c) **Mathematical Induction**: This is a formal proof technique. You prove the claim for the first case. Then prove that if it is true for any given case it is true for the next one. From this we can deduce that it is true for every case. [This explanation is a bit oversimplified.]